GRAYS TUITION CENTRE – Online Tutoring

WEEK: 3

Week Beginning: (04/01/2021)

Subject: MATHS

Year: 10

Lesson Objective:

- Continuing with angles and understanding what the alternate segment theorem is.
- Be able to use all previous circle theorem rules and are able to spot them within questions.
- Practising circle theorem skills and being able to identify them
- Looking into proving all circle theorems that have been taught in all lessons

Class Worksheets

- Page 82, 83, 84, 85, 86, 87, 88 GCSE Maths 4-9 Elmwood (Blue book)

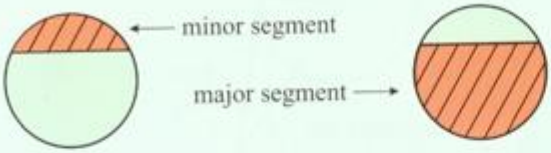
Homework

- Completing classwork for homework
- Revisiting back on surds and making sure its fresh in the mind (build up long term memory) page 20 test yourself unit 1, page 22 mixed examination questions GCSE Maths 4-9 Elmwood (Blue book).

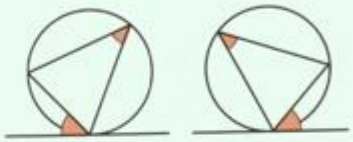
Additional Notes

- All homework from last week will be marked at the beginning of the lesson. Make sure that you have your homework with you in the lesson and are ready to mark it. also prepare any questions if you struggled with the homework so I can help you.
- All questions circled in red are the questions that you are supposed to do.
- All lesson worksheets and **homework for next week (due Week 4)** worksheets can be found below

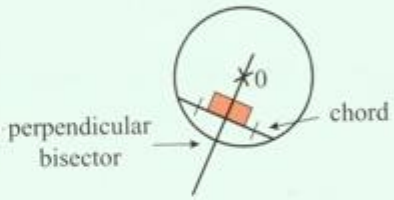
Key Facts



8. The angle between a tangent and a chord is equal to the angle at the circumference in the alternate segment (known as the alternate segment theorem).



9. The perpendicular bisector of a chord passes through the centre of the circle.

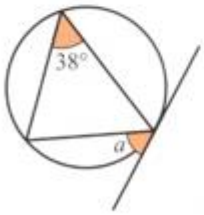


We will prove these rules later in this unit.

E3.4

Find the angles marked with letters.

1



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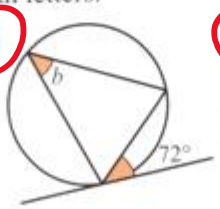

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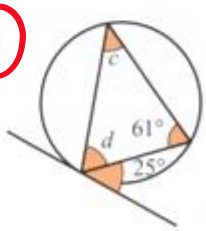

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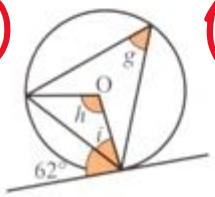

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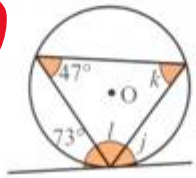

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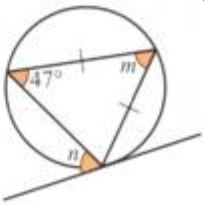

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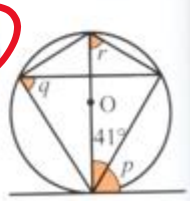

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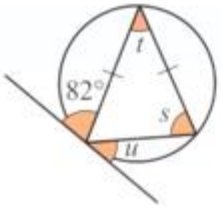

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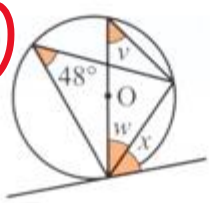

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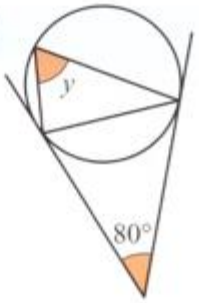

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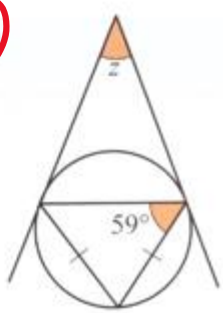

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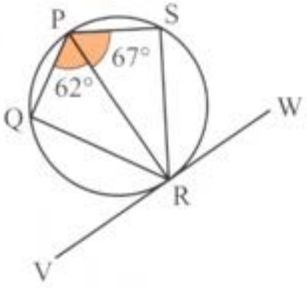


In this question, write down *all the reasons* for your answers.

Find

(a) $S\hat{R}W$ (b) $Q\hat{R}V$ (c) $Q\hat{R}S$

Can you still?

Surds

1. Simplify

(a) $\sqrt{18}$ (b) $\sqrt{45}$ (c) $\sqrt{160}$

2. Simplify

(a) $\sqrt{80} - \sqrt{20}$ (b) $(\sqrt{2} + \sqrt{3})^2$

3. Prove that $\frac{3}{\sqrt{7}} - \frac{2}{\sqrt{10}} = \frac{15\sqrt{7} - 7\sqrt{10}}{35}$

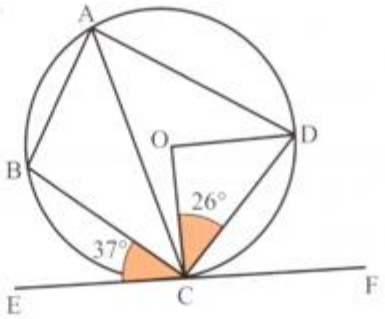
14 In this question, write down *all the reasons* for your answers.

Find

(a) $B\hat{A}C$

(b) $C\hat{O}D$

(c) $B\hat{A}D$



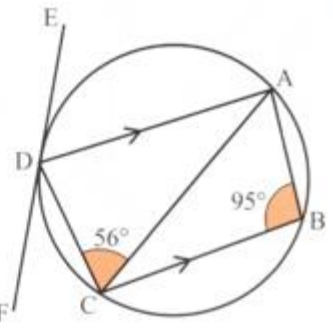
15 In this question, write down *all the reasons* for your answers.

Find

(a) $E\hat{D}A$

(b) $A\hat{D}C$

(c) $A\hat{C}B$



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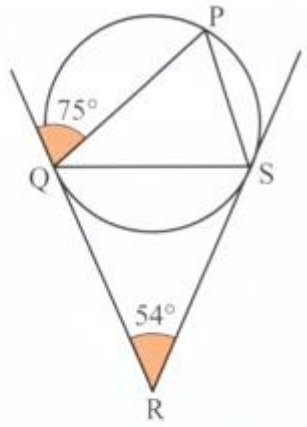


In this question, write down *all the reasons* for your answers.

Find

(a) $Q\hat{S}R$

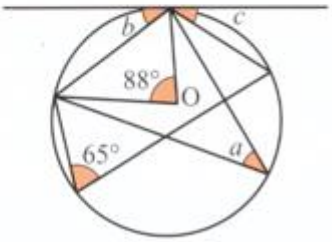
(b) $Q\hat{P}S$

(c) $P\hat{S}Q$

(d) $P\hat{Q}S$

In questions **17** to **21**, find the angles marked with letters.

17



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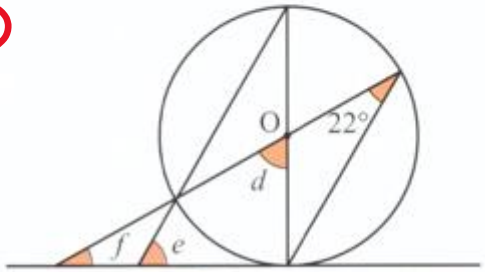

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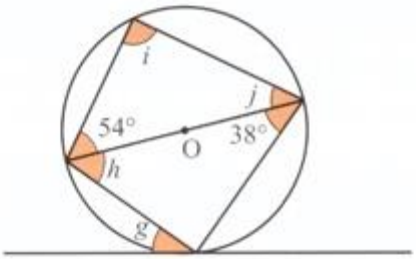

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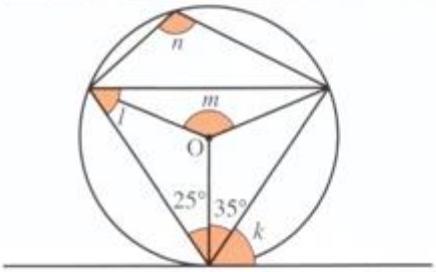

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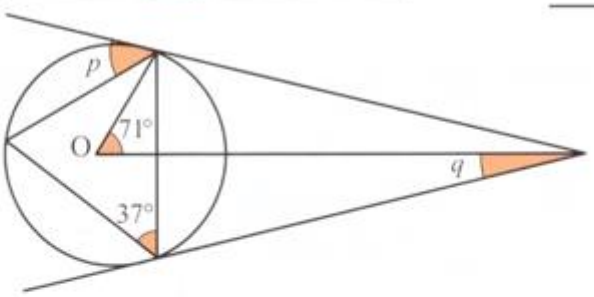

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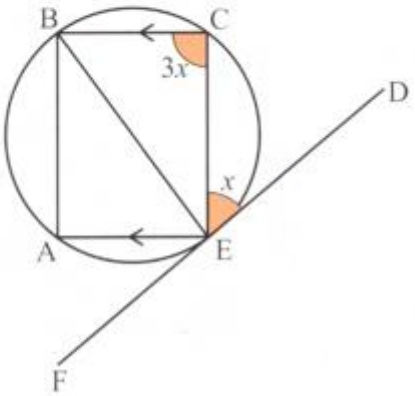
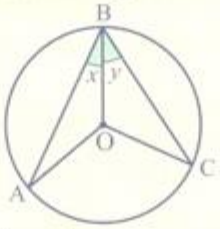


DF is a tangent to the circle at E.
Express angle AEF in terms of x.

Prove that 'the angle at the centre is twice the angle at the circumference in any circle'.

Use algebra

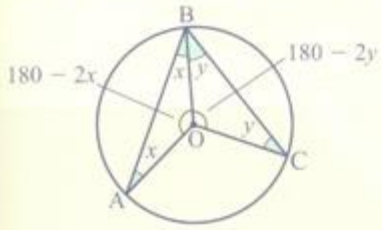


$O\hat{A}B = x$ (triangle OAB is isosceles)

$A\hat{O}B = 180 - 2x$ (sum of angles in a triangle = 180°)

$O\hat{C}B = y$ (triangle OCB is isosceles)

$B\hat{O}C = 180 - 2y$ (sum of angles in a triangle = 180°)

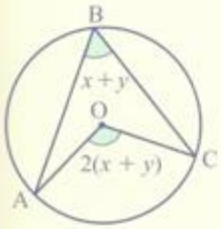


$A\hat{O}C = 360 - (180 - 2x) - (180 - 2y)$

(sum of angles at a point add up to 360°)

$A\hat{O}C = 360 - 180 + 2x - 180 + 2y$

$A\hat{O}C = 2x + 2y = 2(x + y)$



This proves that $A\hat{O}C = 2 \times A\hat{B}C$

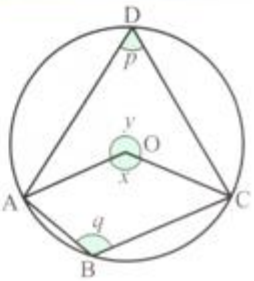
i.e. the angle at the centre is twice the angle at the circumference.

The questions in the Exercise below will guide you through the proofs of many of the circle properties used in this unit. You should become familiar with these proofs for your GCSE exams.

example for lesson

E3.5

1 Copy and complete the following proof that 'opposite angles in a cyclic quadrilateral add up to 180°'.



$x =$ ☐ (angle at centre is twice angle at circumference)

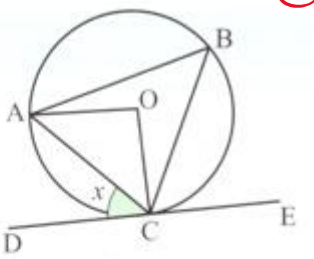
$y = 2q$ (☐)

$x + y =$ ☐ (sum of angles at a point = ☐)

☐ $+ 2q =$ ☐

$p + q =$ ☐ so opposite angles in a cyclic quadrilateral add up to 180°.

example for lesson

2 

Copy and complete the following proof for the alternate segment theorem.

$A\hat{C}O =$ ☐ (angle between tangent and radius is 90°)

$C\hat{A}O =$ ☐ (triangle CAO is isosceles)

$A\hat{O}C = 180° - C\hat{A}O - A\hat{C}O$ (sum of angles in a triangle is 180°)

$A\hat{O}C = 180° - ($☐$) - ($☐$)$

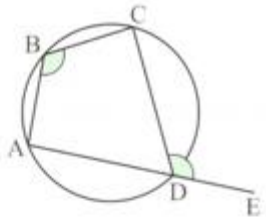
$A\hat{O}C = 180° -$ ☐ $+$ ☐ $-$ ☐ $+$ ☐

$A\hat{O}C =$ ☐

$A\hat{B}C = \frac{1}{2} \times$ ☐ (angle at centre is twice angle at circumference)

$A\hat{B}C =$ ☐ $= A\hat{C}D$ This proves the alternate segment theorem.

3 Prove that $A\hat{B}C = C\hat{D}E$ (i.e. the exterior angle of a cyclic quadrilateral equals the opposite interior angle).



Can you still?

Mixed

Work out

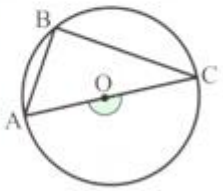
1. $3\frac{1}{8} - 5\frac{2}{3}$ 2. $(-0{\cdot}7)^2$ 3. $2\frac{1}{2} \div (-3\frac{1}{2})$

4. Express $0{\cdot}\dot{5}\dot{1}\dot{3}$ as a fraction.

5. Estimate $\dfrac{\sqrt[3]{28} + \sqrt{24{\cdot}6}}{0{\cdot}503}$

6. Express the ratio $\frac{7}{8} : \frac{2}{7}$ in its simplest form $m : n$ where m and n are integers.

7. Simplify $\sqrt{36a^6b^{10}}$

4 

Copy and complete the following proof that 'the angle in a semi-circle is 90°'.

$A\hat{O}C = 2 \times$ ☐ (angle at centre is ☐ angle at circumference)

$A\hat{O}C =$ ☐ (a straight line)

$A\hat{B}C = \frac{1}{2}$ of ☐

$A\hat{B}C =$ ☐ so the angle in a semi-circle is 90°.

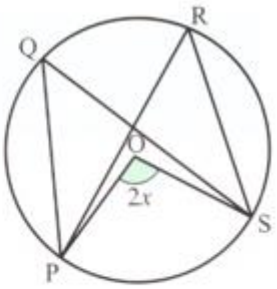
example

example

5 Copy and complete:

$P\hat{Q}S =$ ☐ (angle at centre is twice angle at circumference)

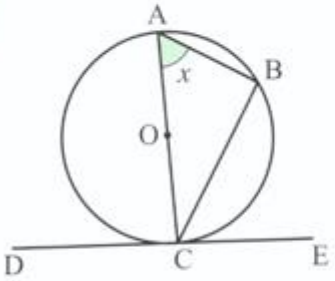
$P\hat{R}S =$ ☐ (angle at centre is ☐ angle at circumference)



so $P\hat{Q}S = P\hat{R}S$ so angles at the circumference are ☐ if the angles stand on the same arc.

example

6



Copy and complete:

$B\hat{C}E =$ ☐ (alternate segment theorem)

$A\hat{B}C =$ ☐ (angle in a semi-circle is ☐)

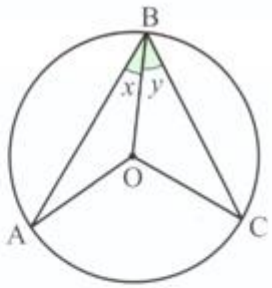
$A\hat{C}B =$ ☐ (sum of angles in a triangle is 180°)

$A\hat{C}E = A\hat{C}B + B\hat{C}E$

$A\hat{C}E =$ ☐ + ☐

$A\hat{C}E =$ ☐ This proves that 'the angle between a tangent and a radius is 90°'.

7 Use this diagram to prove that 'the angle at the centre is twice the angle at the circumference'. (Look at the earlier example if you need to but try to avoid it if possible. You need to learn this proof).



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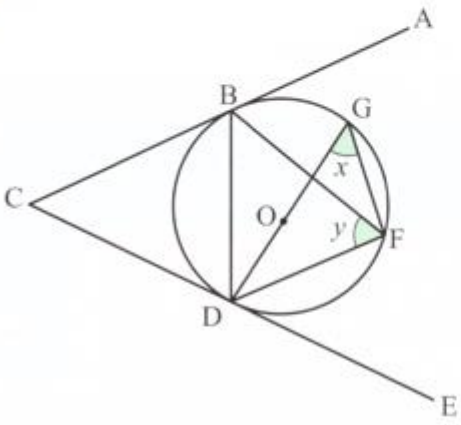


Find the following angles in terms of x and y.

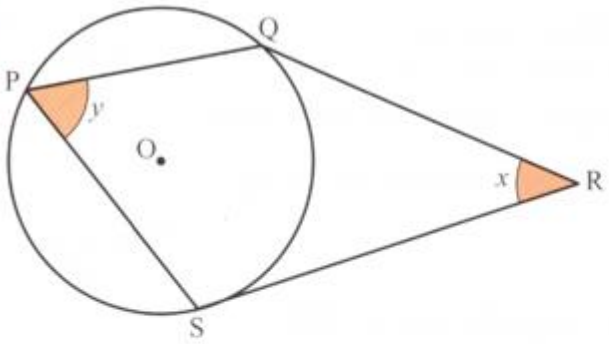
(a) $G\hat{D}F$

(b) $D\hat{B}F$

(c) $C\hat{B}D$

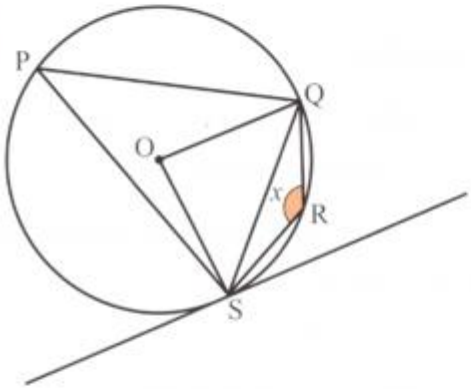
(d) $B\hat{G}F$

9



QR and SR are tangents to the circle.
O is the centre of the circle.
Prove that $y = 90 - \frac{1}{2}x$.

10

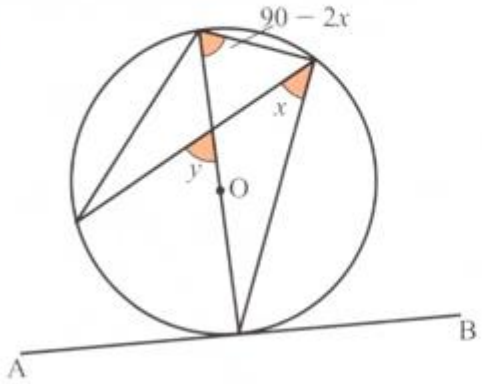


O is the centre of the circle.

Express angle OQS in terms of x.
Give the answer in its simplest form.

Explain your answer fully.

11



O is the centre of the circle.
AB is a tangent to the circle.

Express y in terms of x.
Give the answer in its simplest form.

5. Converting recurring decimals into fractions

Express the following recurring decimals as fractions in their lowest form:

(a) $0.\dot{2}\dot{3}$ (b) $0.4\dot{8}$ (c) $0.\dot{7}1\dot{9}$

6. Simplifying surds

(a) Which of the statements below are true?

(i) $\sqrt{10} - \sqrt{7} = \sqrt{3}$ (ii) $\sqrt{5} + \sqrt{11} = 4$ (iii) $\sqrt{3} \times \sqrt{7} = \sqrt{21}$

(iv) $2 \times \sqrt{5} = \sqrt{10}$ (v) $\sqrt{30} \div \sqrt{6} = \sqrt{5}$ (vi) $(\sqrt{3})^2 = 3$

(b) Simplify as far as possible

(i) $\sqrt{27}$ (ii) $\sqrt{80}$ (iii) $4\sqrt{2} \times 3\sqrt{5}$

(iv) $(4\sqrt{3})^2$ (v) $7\sqrt{2} - 3\sqrt{2}$ (vi) $\sqrt{32} - \sqrt{18}$

7. Expanding brackets containing surds and rationalising denominators

Expand and simplify

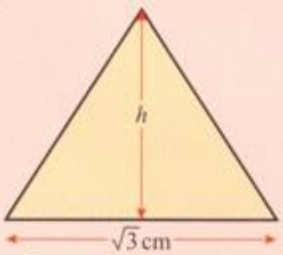
(a) $(\sqrt{2} + \sqrt{5})(\sqrt{3} - \sqrt{2})$ (b) $(1 + \sqrt{3})(\sqrt{5} - 3)$ (c) $(2 + \sqrt{11})(2 - \sqrt{11})$

(d) $\sqrt{5}(2 - \sqrt{3})$ (e) $(1 + \sqrt{3})^2$ (f) $(\sqrt{5} - 7)^2$

Rationalise the denominator in each of the following:

(g) $\frac{1}{\sqrt{11}}$ (h) $\frac{3}{\sqrt{2}}$ (i) $\frac{\sqrt{5}}{\sqrt{7}}$ (j) $\frac{\sqrt{3} - 5}{\sqrt{2}}$

(k)



The area of this triangle is $\sqrt{48}$ cm^2.

Find the height h of the triangle.

(l) Express $\frac{12}{\sqrt{3}} + \frac{6}{\sqrt{12}}$ in the form $a\sqrt{3}$

where a is an integer.

4 Write $\sqrt{12} + \sqrt{75}$ in the form $a\sqrt{3}$ where a is an integer. (AQA)

5 Expand and simplify $(2 + \sqrt{2})(3 + \sqrt{8})$

Give your answer in the form $a + b\sqrt{2}$ where a and b are integers.

(EDEXCEL)

6 (a) Which of the following fractions is nearest in value to $\frac{1}{4}$?

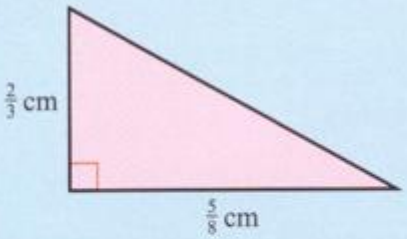
$\frac{3}{10}$ $\frac{7}{20}$ $\frac{4}{15}$ $\frac{17}{60}$

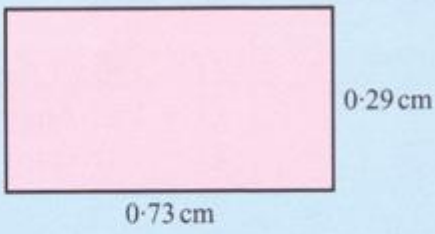
Show your working.

(b) Write down the meaning of $0.\dot{5}\dot{7}$ (CEA)

9 Express $0.7\dot{5}\dot{2}$ as a fraction. (WJEC)

10





Which shape has the greater area and by how much (give the answer to 3 significant figures)?

11 Work out the mean of the three numbers $\sqrt{75}$, $\sqrt{75}$ and $\frac{6}{\sqrt{3}}$

Give your answer in the form $b\sqrt{3}$ where b is an integer. (AQA)

12 Emer says she has worked out that $3 - \sqrt{5}$ is a square root of $14 - 6\sqrt{5}$

Showing all your work clearly prove that Emer is correct. (CEA)

13 At 06:00 hours one day, the temperature in Kiev is $-8\frac{1}{4}$°C. By 11:00 hours the temperature rises by $5\frac{3}{5}$°C. At the same time of day the temperature in London is $3\frac{1}{6}$°C. What is the difference in temperature between Kiev and London at 11:00 hours on this day?

14 Given that $f = \sqrt{2}$, $g = \sqrt{5}$ and $h = \sqrt{10}$, find, in its simplest form,

(i) $\frac{fg}{h}$, (ii) $fg + h$, (iii) fh. (WJEC)