GRAYS TUITION CENTRE – Online Tutoring

WEEK: 15 (updated)

Week Beginning: (29/06/2020)

Subject: MATHS

Year: GCSE

Lesson Objective:

- We will explore trigonometric identities (SOHCAHTOA) and understanding their relationship
- Be able to understand patterns in Sin, Cos and Tan graphs
- Be able to find exact values of trig values and be able to memorise some important values for non calc papers

Class Worksheets

- Page 328 – 330, 546 - 550 GCSE Maths 4-9 Elmwood (Blue book)
- Maths Genie Exercise – Trig and exponential*

Homework

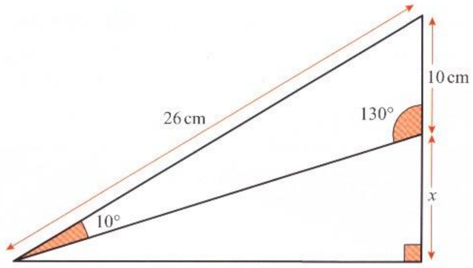
- Complete any remaining classwork for homework and maths genie questions

Additional Notes

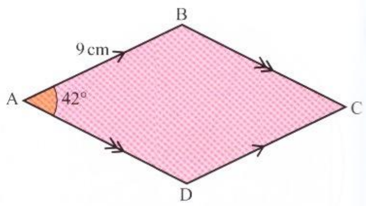
- All homework from last week will be marked at the beginning of the lesson. Make sure that you have your homework with you in the lesson and are ready to mark it
- Also prepare any questions if you struggled with the homework so I can help you.
- All lesson worksheets and homework for next week (**due Week 16**) worksheets can be found below

*https://www.mathsgenie.co.uk/resources/trigandexponential.pdf

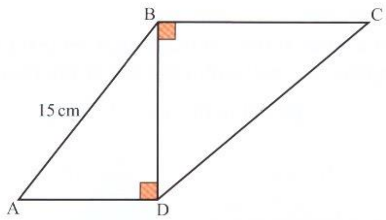
17 Find the length x.



18 ABCD is a rhombus. Find the lengths of the diagonals AC and BD.



19 *Do not use a calculator* in this question.



$\cos A\hat{B}D = \dfrac{3}{5}$

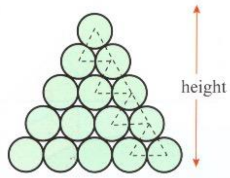
$\sin A\hat{B}D = \dfrac{4}{5}$

$\tan B\hat{C}D = \dfrac{2}{3}$

(a) Work out BD.

(b) Work out BC.

20 15 wine bottles are stacked in a rack. How high is the stack if each bottle has a diameter of 8 cm?



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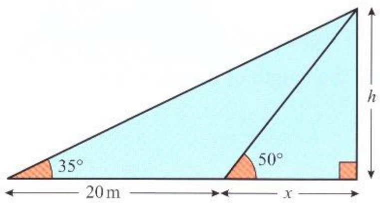
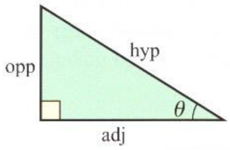


Find x and h.

E Trigonometry for angles of any size

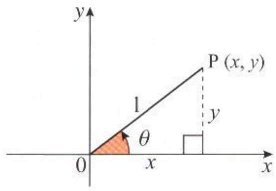


We know

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ and $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

This is true for any angles less than 90° in a right-angled triangle.

We can use the following definition for angles of any size. The co-ordinates of P are (x, y).

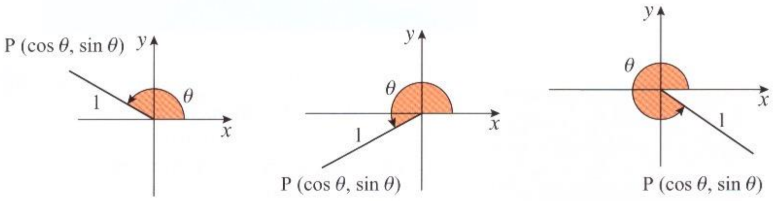


We can see opposite that:

$\cos\theta = \dfrac{x}{1}$ so $x = \cos\theta$

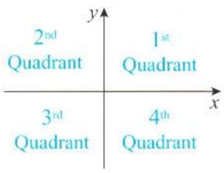
$\sin\theta = \dfrac{y}{1}$ so $y = \sin\theta$

The co-ordinates of P are therefore $(\cos\theta, \sin\theta)$. The angle θ can increase to any size but we define the co-ordinates of P as always being $(\cos\theta, \sin\theta)$.



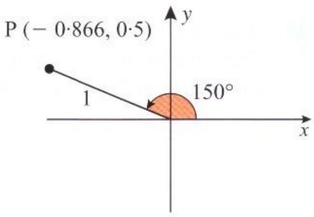
Note – if θ is measured in an anticlockwise direction, it is taken to be positive (θ will be a negative angle if it is measured in a clockwise direction).

Quadrants



Angles between 0° and 90° lie in the 1st quadrant.
Angles between 90° and 180° lie in the 2nd quadrant.
Angles between 180° and 270° lie in the 3rd quadrant.
Angles between 270° and 360° lie in the 4th quadrant.

Consider the following point:



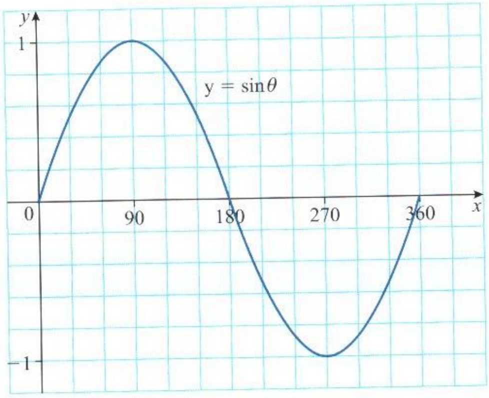
The co-ordinates of P are (cos 150°, sin 150°)

so $\cos 150° = -0{\cdot}866$

and $\sin 150° = 0{\cdot}5$

If θ is obtuse, $\cos\theta$ is always negative.

The values of $\sin\theta$ and $\cos\theta$ can be explored in each quadrant. These values are stored on calculators. These can be used to plot the graphs of $y = \sin\theta$ and $y = \cos\theta$.



We can see that $\sin 150^\circ = \sin 30^\circ$

$\sin 135^\circ = \sin 45^\circ$

$\sin 165^\circ = \sin 15^\circ$

We have $\sin\theta = \sin(180^\circ - \theta)$

You must learn for your exam that:

$\sin 0^\circ = 0$

$\sin 90^\circ = 1$

The sine curve above the x-axis has symmetry about $x = 90^\circ$ and below the x-axis has symmetry about $x = 270^\circ$

E17.1

1 (a) Use a calculator to find the values of $\cos\theta$ for values of θ from 0° to 360° using intervals of 30°.

(b) Draw a graph of $y = \cos\theta$.

(c) Find three different values of θ from your graph which illustrate that

$\cos\theta = -\cos(180^\circ - \theta)$

This relationship is always true.

You must learn for your exam that:

$\cos 0^\circ = 1$

$\cos 90^\circ = 0$

2 Which of the statements below are true?

(a) $\cos 300^\circ = \cos 60^\circ$

(b) $\cos 315^\circ = -\cos 45^\circ$

(c) $\cos 210^\circ = \cos 30^\circ$

(d) $\cos 120^\circ = -\cos 60^\circ$

(e) $\cos 135^\circ = -\cos 45^\circ$

3 Draw your own graph of $y = \sin\theta$ or use the graph drawn earlier to decide which of the statements below are true:

(a) $\sin 210° = \sin 30°$

(b) $\sin 120° = \sin 60°$

(c) $\sin 315° = -\sin 45°$

(d) $\sin 240° = -\sin 60°$

(e) $\sin 300° = \sin 60°$

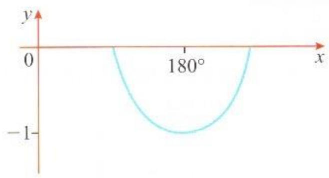
4 (a) Use a calculator to find the values of $\tan\theta$ for values of θ from 0° to 360° using intervals of 20°.

(b) What happens if you try to work out $\tan 90°$ and $\tan 270°$?

(c) Draw a graph of $y = \tan\theta$.
At $x = 90°$ and $x = 270°$, draw vertical dotted lines.
These dotted lines are called *asymptotes*. The tangent curve will never actually cross an asymptote.

Learn for your exam that $\tan 0° = 0$

5

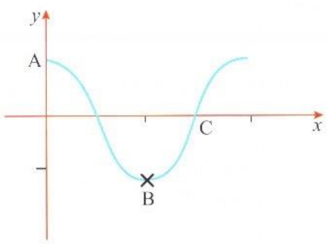


Is this part of the graph of $y = \sin x$ or $y = \cos x$?

6 Which of the following curves pass through the point (180, 0): $y = \sin x$, $y = \cos x$, $y = \tan x$?

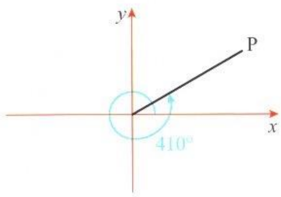
7 Does $\cos x$ ever equal 1·5?

8



This is the graph of $y = \cos x$.
Write down the co-ordinates of the points A, B and C.

9

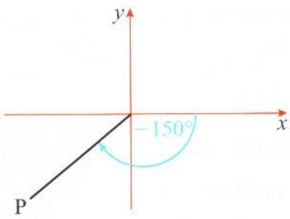


(a) Draw the graph of $y = \sin\theta$ for values of θ from 0° to 720°.

(b) Find three different values of θ from your graph which illustrate that $\sin(360° + \theta) = \sin\theta$.

An angle greater than 360° is obtained by travelling more than one complete turn in an anticlockwise direction.

10



A negative angle is obtained by travelling in a clockwise direction from the x-axis.

(a) Draw the graph of $y = \cos\theta$ for values of θ from $-360°$ to $360°$.

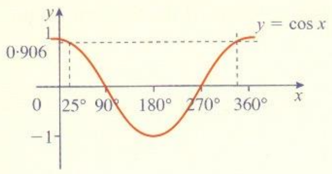
(b) Does the relationship $\cos(360° + \theta) = \cos\theta$ seem to be true?

(c) Draw the graph of $y = \tan\theta$ for values of θ from $-360°$ to $360°$.

(d) Find three different values of θ from your graph which illustrate that $\tan(180° + \theta) = \tan\theta$.

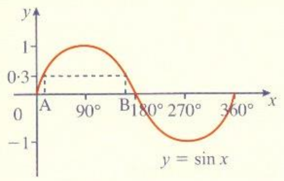
E Solving equations involving sin, cos or tan

(a) If $\cos 25° = 0{\cdot}906$, find another angle whose cosine is $0{\cdot}906$.



By using the symmetry of the graph of $y = \cos x$, another angle whose cosine $= 0{\cdot}906$ is $360° - 25° = 335°$

(b) Solve $\sin x = 0{\cdot}3$ for x-values between $0°$ and $360°$.



SHIFT sin 0·3 on a calculator to find an angle whose sine is $0{\cdot}3$.
INV

We find that $\sin 17{\cdot}5° = 0{\cdot}3$.

correct to 1 decimal place

On the graph, A $= 17{\cdot}5°$. By using symmetry, another angle whose sine $= 0{\cdot}3$ is B which is $180° - 17{\cdot}5° = 162{\cdot}5°$

The solutions of $\sin x = 0{\cdot}3$ in the range $0 \leqslant x \leqslant 360°$ are $x = 17{\cdot}5°$ and $162{\cdot}5°$

E17.2

Use the symmetry of the graphs of $y = \sin x$ and $y = \cos x$ to answer the following questions, giving your answers to the nearest degree.

1 If $\cos 32° = 0{\cdot}848$, find another angle whose cosine is $0{\cdot}848$

2 If $\cos 68° = 0{\cdot}375$, find another angle whose cosine is $0{\cdot}375$

3 If $\sin 18° = 0{\cdot}309$, find another angle whose sine is $0{\cdot}309$

4 If $\sin 230° = -0{\cdot}766$, find another angle whose sine is $-0{\cdot}766$